

## **Determination of Mark-Up Rate under Zero-Interest Financial System: A Microeconomic Approach**

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### **Abstract**

*Misunderstood and maligned, the determinants of mark-up (MU) rate under Murābahah financing deserve scrutiny of its structural formulation. Some suggest that MU rate is really interest rate. Is MU transaction pure trade? We analyze by considering the underlying market structure, central bank imposed minimum reserve requirement, deposit sum, a bank's fixed and variable costs, etc. Perhaps for the first time, the capacity to charge ribā is traced to market imperfection. Banks with no Islamic credential are entering the market suggesting presence of positive economic profit. By promoting proper costing, accountability, efficiency and standardization of MU rate across industry become possible. Prediction and hypothesis testing become possible. Associated with this, we also explore how the deposit rate is determined. The transparency thus afforded should mitigate the contentious debate about MU financing as opposed to interest-based lending.*

Keywords: Mark-up rate, *Murābahah*, ZIFS, Islamic banking.

JEL Classification: G10, G11

KAUJIE Classification : K1, K4

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## 1. Introduction

The massive growth of zero-interest financial system (ZIFS) banks relative to interest-based financial system (IFS) banks is evident from Earnst & Young December 2012 report: ZIFS assets, which “had been growing 50% faster than the overall banking sector assets with an average annual growth of 19% over the past four years, grew to \$1.3 trillion in 2011, and is forecast to grow beyond \$2 trillion by 2014.” The reason for this increasing popularity is put forward very eloquently by the Malaysian Prime Minister, Datuk Seri Najib Tun Razak (2015), speaking at the 11th World Islamic Economic Forum (WIEF),

“More non-Muslims are using Islamic Finance (IF) than Muslims in Malaysia and that is an indication of how widespread the support and acceptance for it is. The popularity of IF around the world was bolstered by the global financial crisis in 2008. The crisis resulted in a sharp demand for alternative economic and business models, specifically financial models which reduced the level of speculation. Unlike the inherent weakness in the conventional model, Islamic finance offered a genuine partnership where both risk and profit is shared. As a result, IF has gained a lot of traction.”

This increasing popularity, support and acceptance have attracted a large number of IFS banks to tap into this ZIFS market. While ZIFS is promising, it has its own share of possible pitfalls. Zahir and Hassan (2001) state that, mark-up (MU) “contracts may open back door to interest. So, while permissible, it should still be restricted or avoided.” While recognizing both MU and PLS (profit-loss-sharing), Chapra (1985), and Kahf and Khan (1992), realize that the former is more likely to violate the underlying religious bidding. Commenting on this issue, Tariqullah Khan (2015) wrote, “We know that *Murābahah*<sup>1</sup> is often abused and turned into *Tawarruq* which creates same level of credit as interest-based lending. The monetary reform movements whether in Switzerland or Iceland, and including IF, can benefit from the potential role of genuine *Murābahah*”<sup>2</sup>.

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<sup>1</sup> Also referred to as Mark-up financing.

<sup>2</sup> Khan wrote: “In a genuine financial *Murābahah*, the bank creates financing only to the extent of the needs of the real markets to finance a car, a house, a project etc. It also makes room for benefiting from the fractional reserve system. However, we know that *Murābahah* is often abused and turned into *Tawarruq* which creates same level of credit as interest-based lending.

*Tawarruq* arises perhaps because loan for the sector with tangible profit is also being extended under MU arrangement (e.g., as with *Istiṣnāʿ*). So, it appears that there is no avenue for direct liquidity

Currently, some secular Muslim and non-Muslim banks from Muslim and some non-Muslim countries (AB Bank of Bangladesh; Lloyds Bank of England; Breitbart; Bank of England) have come forward with “Sharī‘ah” or *ribā*-free window for depositors and borrowers preferring ZIFS. Since interest rate can be easily disguised under the cloak of the MU rate, it raises the question: is there any real difference between MU rate and interest rate in the minds of such newly encroaching entities? Are the two world views one and the same? Or does the assumed mechanics of operation of ZIFS leave room for creative interpretation by profit-seeking or opportunistic non-ZIFS financial institutions? More importantly, is there positive economic profit arising from market power beckoning the newcomers?

In this paper, our purpose is to understand the nature of the MU (or cost-plus) rate in a genuine cost based *Murābahah* system and how this MU rate differs from the interest rate. It is claimed that one of the most important ways interest-based banking differs from MU banking is that while the interest rate is determined by the supply and demand for loanable funds with risk premium added to it at the discretion of the banking system, the MU rate is essentially a cost-based rate.

We address this issue with the expectation that this will produce greater transparency and build confidence about it. It may allow us to better discern the market structure of MU banking. So, we query about the nature of MU rate. How is it formed - what are its determinants and how do they relate to it? How is it affected by PLS investment, if at all? Do operational size, i.e., volume of amount loaned, deposit level, minimum reserve requirement, depositors’ expected rate of return, and bank’s average cost impact the MU rate announced by a ZIFS bank?

Given their differentiating characteristics, the ZIFS banks are likely imperfectly competitive. Regardless, our analysis below is also able to handle market structures of perfect competition, monopoly (by virtue of being the sole operator), natural monopoly (declining AC) and monopolistic competition. To note, ex post, it is possible for both the interest rate and MU rate to take close or same values. That need not be interpreted to mean that there is no difference. On the other hand, the mathematical modeling used here for determining the MU rate could very well be used to parse the interest rate posed by banks. So, could this common foundational approach be the reason that secular banks are opening up interest-free lending window? That, of course, begs the question how, then, is MU rate different from interest or *ribā*? One thing for sure as we figure out the genesis of MU rate, it must be rooted in the cost structure of the ZIFS bank. The lower bound of the MU rate

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insertion to borrowers. This is to be expected when the PLS tool for lending is absent or weak. Therefore, viewing *Tawarruq* as interest bearing loan may be incorrect.

should be determined by the average cost consistent with its definition. After all, the LIBOR – London Interbank Offer Rate - as it stands today is less than 1% (WSJ, 2015). So, although it may be used as a measure of opportunity cost by some ZIFS banks in Muslim countries in calibrating MU rate in a world with ZIFS and IFS banks, it is too low to be the primary determinant of the value that MU rate takes.

On the other hand, currently there is no upper bound of the MU rate on the basis of the argument that the normative economic environment of ZIFS recognizes property right, free and open market, as well as the drive for profit. However, the question arises when, under MU arrangement, an application for a monetary loan made to a ZIFS bank in order to obtain a product is converted into financing of the same product to be sold (traded) by that bank, should the freedom to extract as much profit as the market will bear be followed through? Several points may be made as to why that should not be the case as it otherwise appears to be the current operative assumption. Section II, on Literature Survey, discusses several issues related to this.

In this paper, we try to derive a just MU rate with an upper limit to avoid exploitation and a lower limit to reflect that it is cost-based and hence different from interest rate. While we may idealistically conjecture MU and PLS undertakings of the ZIFS bank to be integrated, we are constrained by the fact that the financial objectives of the two sectors differ substantially. The MU rate is limited by a ceiling on it. The PLS sector is profit driven and is free of any profit restrictions. So, unlike as in Khaled and Khandker [2014], we cannot possibly have one objective function, subject to joint optimization, that includes income and cost flows of both sectors. So, funds flowing into the two sectors have to be separately designated at the time of deposit. After all, the risks faced as well as how the pay-outs are achieved will be different.

Again, as to the deposit rate, since it is determined and distributed post profit flow, it is an endogenous variable, not market determined like the interest rate. So, in economies with ZIFS and IFS banks, everything else being equal, it will not be sufficient to match the year-ending deposit rate with the average annual interest rate in the deposit market. This should address the contention of competing IFS banks that the deposit rate offered by ZIFS banks to their depositors is pre-determined and fixed. Further, announcing an expected deposit rate does not clarify how it was obtained, consequently leaving suspicion as to whether it is mimicking the interest rate in the alternative market. Now, since any deposit in a ZIFS bank is supposed to reflect risk share, should the ex post deposit rate not reflect the gain from that aspect? Further, being risk-sharers in a very unique sense, albeit under *Mudārabah*<sup>3</sup>

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3 Where the depositor cedes to the Bank the right to decide on his behalf what to do with the deposit.

arrangement, should the depositors not have the right to know how much profit was earned and how much was distributed as the return to capital? This will assure depositors that market trend in this regard is being followed, allow assessment of the efficiency of the bank and be enabled to judge the fairness of the share delegated to them. And only then the *Fuqaha*' oversight of such banks will be effective. So, in order to avoid exploitation of depositors in the MU sector, the bank could announce in advance a profit sharing rule between entrepreneurial capital and entrepreneurial effort.

The literature survey is in Section II. Section III deals with definitions and assumptions, where the MU rate is defined and assumptions about the ZIFS bank dealing with MU financing is explored. In Section IV, the actual model of *ribā*-free banking is developed. Before concluding in Section VI, Section V deals with implications of the model.

## 2. Literature Survey

Throughout the ZIFS literature ample references are found about the MU rate. Unfortunately, however, little – if any – allusion exists as to how such a number is calculated. Some tangential reference exists as to the role of opportunity cost, say, LIBOR.

Mark-up is not a new idea in economics. Any firm doing costing and pricing has to face it squarely early in its planning or operational phase. Likewise, the ZIFS banks know it, too. However, as it appears, none of them has come forward and presented formally how it determines the MU rate. Clearly, it is protected, perhaps for business or legal reason, or both. However, as to why academics have not explored this matter is a mystery. That arbitrariness or mistake could lead to *ribā* is a weighty matter not to be ignored.

This secretiveness or lack of attention to an essential detail of ZIFS banking has given fodder to its detractors so much so that many openly claim that the MU rate is simply market interest rate in subterfuge. Thus, the MU rate has a serious PR problem in certain quarters. Such misperception can only be dispelled through adopting a formal, openly bandied about, methodology for this metric. Rhetoric, adjuration, innuendo and assertion will simply not fly.

In this theoretical paper, we extend what we have done over three earlier papers [Khaled and Khandker, 2014, 2015; Khaled, 2015], understand the microeconomics of ZIFS. However, just like other research papers in the field, [Mirakhor (1987) and Siddiqui (2006)], we took the MU rate as given. Khan [1995] recounted the various reasons why PLS has failed to take hold generally, but no exploration of the nature

and limitation of MU financing is tabled. After all, not studying this critical metric at all implies that the *Murābahah* program is not well constructed.

With regard to whether MU financing is pure trade, based on Public Finance literature as well as Faith-based literature, several points are raised. This pertains to ascertaining the upper bound of MU rate.

1. In Public Finance, for example, based on two features that defines a Public Good (Military, Sewer Service, etc.) – Non-excludable (NE; not having to pay upfront) and Non-rival (NR; not having to share) - we identify three other types of goods: Private (E and R; Ice Cream, Cigarette, etc.), Club Good (E and NR; watching a game in a stadium, taking an airline trip, Cable TV, etc.) and Common Resources (NE, R; Ocean Fishing, Environment, etc.). [Mankiw, 2015] In this vein, like Club Goods or Common Resources, any good financed under MU seems to have dual aspects – trade and loan.
2. Our explanation of why MU financing is not a pure trade is based on the Principles of Deductive Law, *Uṣūl al-fiqh* to be precise. [Al-Alwani, 2003] In Islamic Jurisprudence (Sharī‘ah), when no prior ruling exists regarding a novel situation, using the Principle of Analogy (*Qiyās*) could allow application of the old ruling to the new situation. Such a ruling transfer is possible for as long as the core cause (*illah*) of both situations are identical. Thus, for example, the ruling of prohibiting alcohol (*khamr*) applies to drugs as well because of shared *illah*: Like alcohol, drugs disturb the mind and rob it of the capacity to think and act rationally. It can be shown that typical trade (*tejarat*) and “trade” under MU share a common *illah* only partially, at best. That is because the two intentions (*nīyyah*) differ: one seeks to sell a product, while the other, in the guise of a new instrument, seeks to advance money. Further, a bonafide trader has warehouse facility, supply chain, inventory, license, product advertisement, warranty and maintenance plan, service department, etc. A ZIFS bank has none of these trappings. The ZIFS bank, utilizing Qur’ānic verse 2:175, appears to convert a loan into a trade, but it is not a pure trade. As MU financing is a mixture of trade and loan, it is the latter aspect that should restrain MU rate resulting in an upper bound just as *ribā* was constrained on all loans.
3. A few other practical arguments may be raised why the MU rate should have a ceiling even though profit seeking is allowed:
  - a) If a borrower avoids, say, 20% *ribā* on a loan, should he now be required to submit to a MU rate of at least 20% simply because it is a trade being financed? Would that not be exploitation and as a result impermissible?

- b) Would it be justified to not show due regard for the negative perception and dismissive opinion that an unbridled MU rate is likely to produce in the minds of competing traditional IFS banks and potential borrowers? By failing to do so, how damaging would that be in the long run for the industry and the faith of the people in ZIFS? People will eventually see through the smoke. After all, they will get educated and sophisticated!
- c) Finally, as is argued in the paper, the profitability of MU financing for ZIFS banks because of the way it is structured today may be precluding the real test of ZIFS by way of lending under PLS arrangement. If that is the case, then it is a great disservice to the normative position of Islam against *ribā*.

### 3. Definitions and Assumptions

Definitions. In the absence of any testable definition of *ribā*-free mark-up rate, the question arises what is “mark-up” rate that presumably mitigates the incidence of *ribā*? By word choice, mark-up implies “cost-plus” pricing, i.e., an additional amount over and above all costs except return to entrepreneur. So, mark-up is essentially the return for entrepreneurship. While *ribā* is defined as unjust exploitative gain on money lent when it produces excessive or undeserving profit for the business, only the entrepreneur’s worth, i.e., the opportunity cost of entrepreneurship, as accrued by normal profit, is acceptable as the rightful compensation for its service. Abnormal profit implies return to entrepreneur higher than its opportunity cost, and hence, in our definition, constitutes *ribā*. This “ideal definition” of *ribā* leads us to the true *ribā*-free MU rate, which is  $m = AC$  for any amount financed, that produces zero-economic profit. In this formulation, all factors, including capital providers and entrepreneurs, earn their opportunity costs. Under this definition, the mark-up rate could vary among banks should banks tie their individual MU rate to corresponding financed amount.

Now, before proceeding further, two other established methods of determining mark-up can readily be discarded when compared to our definition of *ribā*:

- i. Based on standard Microeconomics of the Firm, our argument suggests that an ability to charge *ribā* can arise only from exercising market power inherent to imperfect competition (monopoly, oligopoly, or monopolistically competitive market structure). Thus, normal practice of setting  $MR = MC$  for profit maximization and choosing a mark-up rate corresponding to the intersection point from the demand curve, would constitute *ribā*, simply because it exceeds the corresponding AC allowing abnormal profit to be reaped.

- ii. Marginal cost pricing, also known as socially desirable or allocatively efficient pricing, will also fail to produce *ribā*-free MU rate under 'normal' monopoly or monopolistic competition (Figure 2) as it will produce abnormal profit. Under natural monopoly, or under monopolistic competition as described below in Situation I, marginal cost pricing, on the other hand, will produce an economic loss and hence will not be sustainable.

Below, three evolving market situations under imperfect competition are explored to see how the break-even rule,  $m = AC$ , could play out.

Situation I: The falling segment of the AC curve intersects the demand curve for MU financing (AR). This is the likely short-run situation for a start-up ZIFS bank or a ZIFS bank with a small market size or market share under monopolistic competition. Its business presence in the area as well as its expertise are not yet quite known and endorsed. So, the demand is waiting to expand over a period of time as name recognition takes hold. Figure 1 represents this situation<sup>4</sup>. Below, 'L' is defined as the amount of loanable funds actually invested in the MU sector. Where the AC curve intersects the AR curve,  $m = AC$  rule will produce a shortage of funds for all available loanable funds less than  $L^*$ . Consequently,  $m^*$  is the break-even MU rate corresponding to  $L^*$ .<sup>5</sup>

Situation II: The rising segment of the AC curve intersects the demand curve for MU financing (AR). This is likely the situation when the ZIFS bank has matured somewhat in its business. Figure 2 represents this situation. This diagram corresponds to the normal case of a monopolistically competitive firm in the short-run. It also represents the case of a monopoly where only one firm dominates the market possibly by virtue of being the first mover.

Situation III: The falling segment of the AC curve is tangent to the demand curve for MU financing (AR). This is the situation of a monopolistically competitive firm in the long-run when the market for ZIFS banks has matured and stabilized. Figure 3 represents this situation.

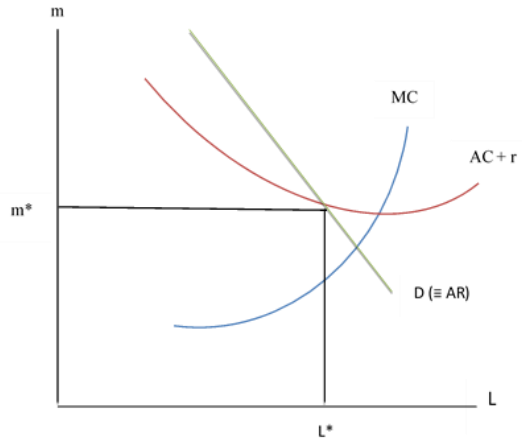
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<sup>4</sup> This can also be the case of a natural monopoly.

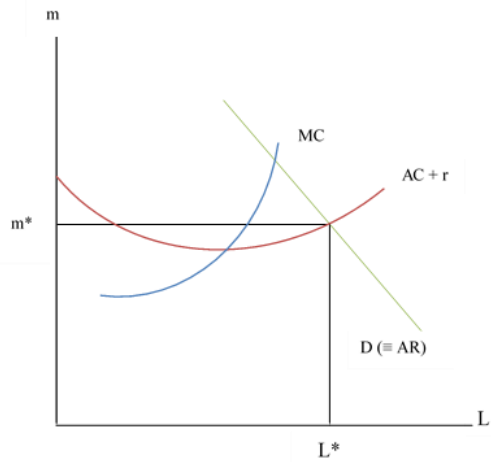
<sup>5</sup> In case the AC curve intersects AR at two different points, the higher L will be associated with a lower m.



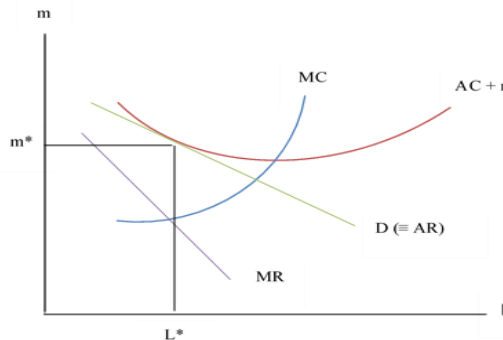
**Figure 1**  
**Monopolistic Competition, Short-run Start-up Phase**



**Figure 2**  
**Monopolistic Competition, Short-run Competition Evolving**



**Figure 3**  
**Monopolistic Competition, Long-run Mature and Stable**



For Situations I and II,  $m$  is determined by the amount financed,  $L$ , that the bank is willing and able to finance under our “ideal definition” of *ribā*-free MU rate. Here,  $m$  is determined by the individual banks. It would be free to set its own ‘ $m$ ’ anywhere on the  $AC$  curve from its point of intersection with the demand curve (i.e., where  $AC = AR$ ) moving to the left. At any such point, the bank would be breaking even, i.e., profit would be same. So, no matter the solution  $(L, m)$ , the bank would be able to extract its opportunity cost and no more. Based on this, at least two options are possible for a “not for-profit” bank:

- i. Finance using all of available  $L$  up to the point  $L^*$  where  $AR = AC$ . That would mean maximizing the number of borrowers served. In this scenario, should the amount of money available to the bank fall short of  $L^*$ , then  $m$  will settle on  $AC$  at the point matching this sum.
- ii. Finance to the point where  $m$  is at its minimum, i.e., when  $AC$  bottoms out. That would mean providing the cheapest financing possible to best serve the community.

As for the bank that, driven by market opportunity, charges a MU rate greater than  $m = AC$ , this alternative solution may pose a problem. Such a “for-profit” bank, unlike the not-for-profit bank, will tend to reduce the amount of  $L$  financed, as extending a greater amount only increases work-effort without impacting profit. In a world with banks striving to maximize their profit, this is a serious limitation brought on by the “ideal definition” of *ribā*-free mark-up. As a result, we could “compromise” our ideal definition with a “second best” option. In this situation, the

central bank would set  $m^* = AC = AR$  as the maximum possible MU rate. This has both standardization and regulatory aspects to it. The not-for-profit banks may still go for solution (i) or (ii). However, for the for-profit banks, this opens the scope to reap abnormal profit in a limited way while still adhering to a global *ribā*-free MU rate. It will, in this case, finance  $L < L^*$  where  $m^* = MC$  to maximize its profit and reap some acceptable abnormal profit.

Assumptions. When a ZIFS bank is dealing with two essentially non-substitute products: MU and PLS financing, its optimization problem has different characteristics. These two types of financing differ in their degree of riskiness. MU financing is associated with minimal risk compared to PLS financing. The characteristics of this bank can be portrayed through the following assumptions.

- i. Risk averters will choose to deposit their money in the transactions account which can only be invested for MU financing. Because of the risk associated with it, transaction account funds cannot be invested for PLS financing. Investment account depositors, on the other hand, are willing to take higher risk for higher profit. Their funds, however, can be invested in projects, both in PLS and MU sectors, where they earn highest profit. Thus, the objective functions and the associated optimization routines will be separate and different.
- ii. This characteristic of the investment funds ensures that, while no excess fund may flow from MU to PLS, funds could flow from PLS to MU whenever marginal dollar earns higher return from MU investment.
- iii. While the ZIFS bank has a fiduciary responsibility toward the depositors, and also a business imperative to share profit to attract and retain them, there is no legal promise of any payment whatsoever except to share in profits earned. This may involve revealing the adopted profit sharing formula between capital and entrepreneur for both groups of depositors.
- iv. Outside of the optimization routine, there is a risk-share arrangement between the bank and its clients. This may affect the nature of the problems related to Adverse Selection and Moral Hazard (both unexplored here) which depositors face with respect to the bank and the bank faces with respect to both species of borrowers.
- v. MU and PLS activities are viewed as two separate divisions of the same firm. Here, while the revenues earned are distinct, the costs incurred by these two entities are not completely separate. A pro-rated system is used to assign costs appropriately.

#### 4. Model

$T_A$  = Total customer deposit that can be used only in MU sector (Transactions Account)

$T_I$  = Total customer deposit that can be used in both MU and PLS sectors (Investment Account)

$C$  = Capital amount invested in the bank by the owners of the bank

$E$  = Opportunity cost for entrepreneur's time and effort

$\rho$  = Opportunity cost of  $C$  based on return on second-best investment opportunity ( $\rho > 0$ )

$d$  = Expected (ex-ante) market rate of return to depositors

$TFC = (dT_A + \rho C + E) + OFC$  = Implicit fixed costs (payables to depositors, capital owners and entrepreneurs) + Other fixed costs for MU and PLS activities

$TVC_M$  = Total variable cost of MU investment

$TC_M$  = Total cost of MU investment =  $[TVC_M + dT_A + T_A/(T_A + T_I)(\rho C + E + OFC)]$

$K$  = Bank owned cash available for MU financing, where  $K < C$

$\sigma$  = Required Reserve ratio on  $T_A$ , ( $0 \leq \sigma < 1$ )

$(1 - \sigma)T_A + K$  = Loanable funds available to the bank for MU investment

$L$  = Amount of loanable funds actually invested in MU sector =  $\alpha\{(1 - \sigma)T_A + K\}$ ,  $0 \leq \alpha \leq 1$

$r$  = Fraction of losses to investment as a moving average for a chosen number of prior years

$m$  = Mark-up rate bank charges on MU investment

Below, for determining the *ribā*-free 'm' under imperfect competition, we set out the breakeven relationship between total revenue and total cost for the bank. To simplify, the principal sum financed,  $L$ , has been deducted from both sides of the equality.

Further, like most other businesses, financing undertaken by ZIFS banks is not risk-free. A portion of the monies financed is lost when the borrower fails in his project thereby becoming unable to service his debt. So, what can the bank do to protect itself? When such an event occurs, the bank stands to lose on two fronts:

unpaid principle and unpaid MU earnings. We use a multi-year moving average of losses and monies financed in previous years to represent the fraction of loss suffered. This parameter is applied to current expected total amount financed to estimate total expected loss, which is then used to calculate net total revenue.

Given the earlier definition of  $m$ , for breaking even,

$$\text{Net TR} = \text{TC}_M$$

$$\text{Or, } (m - r)L = [\text{TVC}_M + \{dT_A + T_A/(T_A + T_I)(\rho C + E + \text{OFC})\}]$$

$$\text{Or, } m = \text{TC}_M/L + r$$

For *ribā*-free  $m$ , we have:

$$m = \text{TC}_M/L + r = \text{AC} + r \quad (1.0)$$

Now, any point on the AC curve that satisfies equation (1.0), under the circumstances described in situations I – III, would give our ideal *ribā*-free MU rate. However, our “second best” definition of *ribā*-free MU rate gives:

$$m^* = \text{AC} + r = \text{AR} \quad (1.0.1)$$

Comparative Static Analysis. Below are the derivatives of  $m$  from Equation (1.0) with respect to the underlying variables.

$$\delta m / \delta d = T_A / L > 0 \quad (1.1)$$

$$\delta m / \delta \rho = CT_A / (T_A + T_I) L > 0 \quad (1.2)$$

$$\delta m / \delta C = \rho T_A / (T_A + T_I) L > 0 \quad (1.3)$$

$$\delta m / \delta E = T_A / (T_A + T_I) L > 0 \quad (1.4)$$

$$\delta m / \delta \text{TVC}_M = 1 / L > 0 \quad (1.5)$$

$$\delta m / \delta \text{OFC} = T_A / (T_A + T_I) L > 0 \quad (1.6)$$

$$\delta m / \delta r = 1 > 0 \quad (1.7)$$

$$\delta m / \delta \sigma = \alpha \text{TC}_M \cdot T_A / L^2 > 0 \quad (1.8)$$

$$\delta m / \delta T_A = [L\{d + (\rho C + E + \text{OFC})T_I / (T_A + T_I)^2\} - \text{TC}_M \alpha(1 - \sigma)] / L^2 \geq < 0 \quad (1.9)$$

$$\delta m / \delta T_I = -T_A(\rho C + E + \text{OFC}) / \{(T_A + T_I)2L\} < 0 \quad (1.10)$$

$$\delta m / \delta L = -\text{TC}_M / L^2 < 0 \quad (1.11)$$

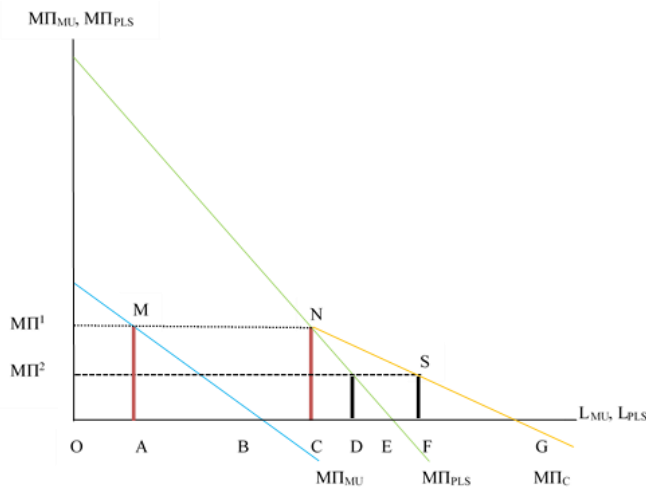
First Order Conditions Interpreted. Since the MU rate,  $m$ , is defined as mark-up above the average cost of financing, an increase in the cost of financing will definitely increase the MU rate. Equations (1.1) – (1.7) reflect this result. Equation

(1.8) says that as the minimum reserve requirement,  $\sigma$ , increases, so does the MU rate. This is because an increase in  $\sigma$  will allow fewer funds available for financing. Since banks have to pay depositors for all of the deposited funds irrespective of whether they are used for financing or not, an increase in  $\sigma$  will increase the average cost of financing. An increase in transactions account deposit,  $T_A$ , on the other hand, has an indeterminate effect on the MU rate, depending on the level of  $L$ , as shown by inequality (1.9). MU rate will fall as  $T_A$  increases if  $L$  is below, will rise if above, and remain constant at the minimum AC financing amount of  $L$ . Finally, inequality (1.10) indicates that an increase in investment deposit will reduce the fixed cost of financing MU investment by shifting a part of the fixed cost to PLS financing, thereby reducing the MU rate. Finally, (1.11) indicates the MU rate falls as the amount of available loanable fund actually invested in the MU sector rises

### 5. Implications of the Model

Inter-sectoral Allocation of Fund. In our model, we made the point that a ZIFS bank has or can have two items in its portfolio: MU and PLS investments. We also explained that the two deposit accounts, TA and TI, are dedicated to these two ends. Point was also made that, compared to PLS, the risk on MU was lower and that the bank, in order to maximize profit, could divert TI toward financing MU investment. We have also established the rate of return on MU investment as the MU rate,  $m$ . This is based on zero economic profit accrual. The PLS investment has no such limit. So, the question then is: under what circumstance would TI be diverted toward financing MU portfolio?

**Figure 4**  
**Possible Reallocation of Fund From PLS Sector to MU Sector**



Suppose we designate the marginal profit of PLS investment by  $MII_{PLS}$ . When total profit is rising at a diminishing rate,  $MII_{PLS}$  is falling, and eventually when total profit reaches its maximum,  $MII_{PLS} = 0$ . Thus,  $MII_{PLS}$  diminishes as PLS investment increases. Similarly, when MU sector can accrue some profit, marginal profit in the MU sector,  $MII_{MU}$ , also diminishes as MU investment increases. As long as  $MII_{PLS} \geq MII_{MU}$ , none of the  $T_I$  will be deployed for financing MU investment. Profit maximization rule might dictate banks to move funds from MU to PLS sector, however, restrictions on  $T_A$  will prohibit banks to do so.

In Figure 4, we explore the possibility of transfer or reallocation of funds,  $T_I$ , from the PLS sector to the MU sector. By assumption, none of the MU fund,  $T_A$ , will transfer in the other direction. First, we construct diminishing marginal profit functions  $MII_{MU}$  passing through M and B and  $MII_{PLS}$  passing through N and E for the MU and PLS sectors, respectively. When loanable fund in MU sector,  $L_{MU}$ , is at least equal to OB, then profit will be maximized in MU sector allocating OB amount to MU investments. At that time, no scope arises for any transfer of  $T_I$  to the MU sector.

Now, should  $L_{MU}$  fall short and be only equal to OA amount, say, then the circumstances under which some of  $T_I$  fund could move to MU sector are explored below. We construct another line, NG, which is the horizontal summation of NE and the difference between MB and MA, and designate it as  $MII_C$ . With OA amount of fund in MU sector, the  $MII_{MU}$  at OA investment is equal to  $MII^1$ . No amount of fund will move from  $T_I$  to MU sector as long as  $T_I \leq OC$  since  $MII_{PLS} \geq MII^1$ . However, when  $T_I > OC$ , say OF,  $MII_{PLS} < MII^1$ . With both sectors present, marginal dollar invested in the MU sector is more profitable than it is in the PLS sector.<sup>6</sup> Since joint profit is maximized when marginal profits in the two sectors are equal, NG line will help us determine the transfer of  $T_I$  funds into MU sector. With funds  $T_I = OF$ , equality of marginal profit in two sectors can be achieved at  $MII^2$  determined by the vertical line FS. Hence, for joint profit maximization, DF will be transferred to the MU sector while retaining OD in the PLS sector. Incidentally, the more elastic is the  $MP_{MU}$  curve, the larger is the amount of transfer.

Reflection on Deposit Rate. Now, the MU rate,  $m$ , was derived by taking  $d$  as given. So, we turn to ex-post profitability and residual distribution among depositors and entrepreneur's own capital and entrepreneurship. With normal economic profit, the corresponding per unit distribution of residual would be as planned:  $d$ ,  $\rho$  and  $E$ . Even under this situation, how the value of  $d$  is picked needs examination for under-paying depositors is as much inequitable as over-charging borrowers. It is clear that

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6 In the absence of a MU sector, profit would be maximized by investing  $OE < OF$  in the PLS sector keeping EF unutilized.

given  $m$ ,  $d$  is inversely related to both  $\rho$  and  $E$ . Of course, the depositors entrust their monies to the bank under *Muḍārabah* arrangement. That means the bank is free to decide how to use those monies and to choose the value of  $d$ . To the extent  $d$  is market driven and that market is imperfect,  $d$  is likely to settle favoring the bank.

If the bank earns positive economic profit, there will be a two stage distribution of such profit. In the first stage, the distribution will be as under normal profit distribution outlined above. The leftover sum of the residual could be subject to a pre-existing rule. Should the suppliers of fund (depositors and entrepreneurs) and the entrepreneur be considered as equal partners, then this additional sum will be shared on a 50-50 and pro-rated basis. In other words, say, with \$1m positive economic profit, \$0.5m would go the bank for its entrepreneurship. Of the remaining sum, should depositors provide 80% of the bank's ( $T_A + K$ ), then \$400,000 would accrue to them in the form of enhanced  $d$ .

On the other hand, with a short-run economic loss, the pre-arranged distribution of economic loss could actually reward the depositors below the projected market rate  $d$ . In that case, the bank may have to let go some of its own income from  $\rho$ ,  $E$ , or both so as to reward depositors as planned in order to remain competitive. The bank, in this case, absorbs the loss with the expectation that the desired flow of deposit will not be disrupted.

Special Case #1 - No Depositors and only MU Financing (Housing Market). Suppose, a financing institution gathers its loanable fund not from depositors but from an outside investing firm which expects a fixed periodic payment<sup>7</sup>. Also, the investing firm having sanctioned a sum of money and created an account for the financing company expects the latter to pay only when it draws down an amount,  $T_A$ , from that account. However, this  $T_A$  is not subject to minimum reserve requirement, i.e.,  $\sigma = 0$ . Also, the investing company expects a fixed, upfront payment rate,  $a$ , on its money. As the financing institution does not engage in any PLS investment or elicit corresponding deposit,  $T_I = 0$ . So, the relevant MU rate under  $m = AC$  rule would be a special case of Equation (1.0):

$$m = [TVC_M + aT_A + \{\rho C + E + OFC\}] / L + r \quad (2.0)$$

Interestingly, to what extent does the typical mortgage rate in affected communities in the US or the UK, for example, differ from that suggested by Equation (2.0) above?

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<sup>7</sup> It is a possibility in the west, or may already be happening by some anecdotal reports, in the name of Islamic financing in (Muslim segment of) the housing industry.



Special Case #2 - Leasing Market for Capital Equipment. If, as in special case #1 above, the loanable fund,  $L$ , is partially gathered, as  $T_A$ , from an outside investing company, but which does not seek a fixed periodic payment, the mark-up rate will affect the rate of return to investing company. Now, unlike as in Equation (2.0), but similar to Equation (1.0), the cost to the financing company is a fixed cost depending on the volume of outstanding financing processed, where  $L = T_A + K$ . Also, there is no PLS activity, and the ZIFS bank gives the same implicit value,  $\rho$ , to its own money as it does to that of outside investing company. So, again, under  $m = AC$  rule, we get:

$$m = [TVC_M + \{\rho(T_A + C) + E + OFC\}]/L + r \quad (3.0)$$

Equation (3.0), also, awaits empirical verification.

## 6. Conclusions

The goal here was to mathematically derive the *ribā*-free MU rate logically consistent with the literature that most likely would be representative of heretofore unknown formula of the MU rate as employed in *Murābahah* financing. Here, as with the rest, the overall strategy has been to formalize the theory for a clearer grasp of the issues widely discussed in the literature, to derive optimization rules wherever necessary, draw up testable hypothesis, and to make prediction possible while creating elasticity measures.

In the particular case explored here, very little theoretical discussion, if any, exists. The MU rate's presence has been universally taken as given without questioning how it could come about. Also, unexplored is the fact that beyond a thorough MU rate determination process, there remains a scope for *ribā* by overcharging the borrowers under the existence of monopoly power. So, a real opportunity to fill in the blank presented itself. We have resorted to standard microeconomics on costing and pricing to get this work done. Further, there is no analysis how the deposit rate is determined and whether its current practice may be extracting reverse-*ribā* by underpaying the depositors under all form of "monopsony-type" market arrangement.

Now, for the first time in the industry, based on our interpretation of the *ribā*-free MU rate, we are able to empirically evaluate the true nature of the existing MU rate. Our assumption that the MU financing market is imperfect is also a testable premise. By suggesting MU market to be imperfect, we bring up the point that the MU sector, as it exists today, may actually be enjoying abnormal profit while under nominal condition of risk thereby causing the riskier PLS sector to be shut out. This is an altogether novel reason why the PLS sector has not caught on. Since the suggested AC-based MU rate reduces economic profit to near zero, it will encourage banks to

pursue more PLS investment in search of higher profits: a result highly desirable in this industry. On the other hand, if banks get completely out of MU investment and pursue PLS investment only, restrictions may be imposed by the central bank that only banks that subject themselves socially and ethically to this formulation will have the opportunity to participate in the riskier but more profitable PLS sector.

The first order conditions allow for predicting sign and magnitude of change in the MU rate owing to changes in underlying variables. Also, corresponding elasticity formulations are readily possible, which lead to macroeconomic management implications.

Also, being able to assess the nature of financing residential houses as well as financing leasing of industrial equipment significantly extends the assessment of *Murābahah* financing as it exists today.

Finally, a couple of interesting systemic implications may be observed on account of prohibition of *ribā* that appear to have a modern day aspect to them. First, without a doubt, the concept of market structure was absent among the Arabs when the Qur'ān was revealed. However, in our effort, we have conjectured that a situation of imperfect market must have existed in order for there to be *ribā*. So, the Qur'ānic prohibition against *ribā* anticipates modern regulatory structures applied to a supplier with highly concentrated market presence. Further, just as in *Suratul Ma'un* [Asad] where the quality underlying the practice of formal spiritualism is connected with real acts of goodness, even those appearing to be like small favors, the imposition against *ribā* is suggestive of the modern understanding that money is like blood to the economy and that what money does or leaves undone in the real sector is of actual import. Thus, again, even though the Arabs of the prophetic era lacked sophisticated economic concepts, the prohibition of *ribā* appears to make the normative point that the nominal sector should be treated as being subservient to the real sector.

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