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Portfolio Determination of A Zero-Interest Financial System Entity

SHAFI A. KHALED•

A. WAHHAB KHANDKER♦

Abstract

Beginning in the 1930s and increasing significantly post-colonialism, some Muslims scholars have wondered about the divine edict of interest-free nominal sector and how to realize it in the context of modern mass deposit institution and wide, concentrated financing demand for trade, entrepreneurship and consumption ends. This undertaking has faced challenges posed by the interest-based nominal sector. Evidence has mounted about the limitations of interest-free banks in the way they are organized and, of late, the largely a theoretical way they do business and their business and political operative environment. For explaining the phenomenon and predicting events, a risk-discounted, expected profit objective function produces rules for inter and intra-sectoral allocation of funds. The non-homogeneity of mark-up and profit-loss-sharing products leads to adopting the average sizes of outlays in the two sectors as the choice variables. Identifying allocation rules for resources will benefit empirical analysis, banking policy and the central bank's monitoring effort.

Keywords: Islamic Finance, Interest, *Ribā*, Diversification, Portfolio, Risk Return, *Mushārakah*, *Muḍārabah*, *Murābaḥah*, Mark-up, PLS
JEL Classification: G10, G11
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• 300 St. John's Hall, 700 East 7th Street, Metropolitan State University, St. Paul, MN 55106, shafi.khaled@gmail.com

♦ Department of Economics, 403Y Carl Wimberly Hall, University of Wisconsin - La Crosse, La Crosse, WI 54601, wkhandker@uwlax.edu

1. Introduction

The interest-based financial system (IFS) and zero-interest financial system (ZIFS)[1] are two financial systems currently prevalent in the world. Since the 1980s, inspired by divine authority [Deuteronomy-23:19[2], Psalms-15:5 (WEB)[3]; Quran-2:275 (Asad)[4]], political reconfiguration and evolving financial clout, there has been a large-scale growth in ZIFS as a parallel system in parts of Africa, the Middle East and South and South East Asia. Although IFS is overwhelming in the lending market, according to Earnst & Young December 2012 report, ZIFS assets, which had been growing 50% faster than the overall banking sector assets with an average annual growth of 19% over the past four years, grew to $1.3 trillion in 2011, and is forecast to grow beyond $2 trillion by 2014.

So far as the IFS is concerned, a group of scholars view it as unjust (reward of a project is distributed in an inequitable manner first between the depositors and the bank, and then the bank and the borrowers; while the nominal factor's unearned income is guaranteed upfront payment much like real factors, the risks are being mainly borne by the borrower.) and inefficient (creditworthiness is stressed over promise of project productivity). They also believe that IFS is detrimental to stability during cyclical downturn when liabilities tend to exceed assets. Further, IFS creates money disregarding the potential end results of failed projects on inflation and business cycle fluctuations that affect society's welfare. However, as to ZIFS, even with many adherents as well as "successes"[5], the debate rages as to what constitutes it, whether it is operating per expectations and how efficient it has been.

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[1] Editor's note: The authors have used ZIFS to underscore its non-denominational, universal scope. However, Islamic finance is more than zero interest financial system.

[2] You shall not lend on interest to your brother; interest of money, interest of food, interest of anything that is lent on interest.

[3] He who doesn't lend out his money for usury nor take a bribe against the innocent, He who does these things shall never be shaken.

[4] Those who gorge themselves on usury behave but as he might behave whom Satan has confounded with his touch; for they say, "Buying and selling is but a kind of usury" - the while God has made buying and selling lawful and usury unlawful. Hence, whoever becomes aware of his Sustainer's admonition, and thereupon desists [from usury], may keep his past gains, and it will be for God to judge him; but as for those who return to it -they are destined for the fire, therein to abide!

[5] The empirical literature does find it wanting in many regards.

Regarding ZIFS, no doubt, there has been sizable comparative empirical evaluation of its practices, but one issue has gone unattended. While theoretical points have been made, there has been a lack of systematic economic modeling of this idea. Thus, the considerations, concerns and criticisms aside, other than applied banking activities conducted on an ad hoc theoretical basis, there is yet to be a formal understanding of the underlying objective function of the ZIFS entity and the associated conditions for optimization. It is not clear, perhaps due to modeling asymmetry that the typical marginal rules of optimization for a firm will apply here. So, no theoretically grounded comparative static analysis is possible that will guide bankers, researchers, and policy makers to assess whether the ZIFS is functioning efficiently. We hope to bring some technical order to this profoundly important but nascent subject matter.

In section 2, we give a brief literature survey relevant to our work. In section 3, we present our methodology and model, while before concluding in Section 5, Section 4 shows the results of our analysis.

2. Literature Survey

ZIFS approves two broad modes of financing. The most desirable (Siddiqi, 1988; Khan, 1992; Mirakhor, 1987; and Ahmed, 1985) and profitable one is the profit-loss-sharing (PLS) equity financing – *Muḍārabah* and *Mushārakah* - where return on capital depends on productivity, and allocation of funds is based on quality of the project rather than the credit worthiness of the borrower (Zaher and Hassan, 2001). It also approves mark-up (MU) financing – *Murābaḥah* – where an existing tangible asset is initially purchased by the bank at the request of the credit-seeking buyer and then resold to the buyer with a cost-plus profit on a deferred sale basis - where legal ownership is transferred following the last payment to the bank. An alternative version of this – *Istiṣnāʿ* - would be when, under a similar credit arrangement, "large", currently non-existent, tangible asset is ordered to be produced, such as a power plant or a ship.

Multiple theoretical and empirical analyses investigate the sectoral distribution of funds between MU and PLS. Khan (1995) documents that in financing investments MU has dominated PLS in spite of the prospect of higher profitability of PLS. Ahmed (2002) refers to similar finding. Contrary to the common expectations, this has been the case shortly after ZIFS financing was introduced. Khan (1983) explained this with the moral hazard hypothesis (MHH) which stipulates that the existence of an economic incentive not to report actual profit for personal gain leads to financial loss of the banks under PLS. As an instance of MHH

for Nigeria, Aburime and Alio (2009) cite tax avoidance as one reason for underreporting earnings by investors, while small businesses are shoddy at record keeping and large businesses understandably want to keep their financial accounts private. This, of course, disadvantages the ZIFS bank. So far MHH was the dominant reasoning accepted in the literature for the explanation of the dominance of MU system in ZIFS (Tag El-din, 1991; Siddiqi, 1988, 1993; al Qari, 1993). Khan (1995) argues that if MHH is the only reason for the dominance of MU financing, then the problem can very well be solved by designing a diminishing PLS contract where entrepreneur can buy the project out of the profits generated by the project, a share which has been heretofore fixed and permanent. Now, a permanent partnership would appear to be a backdoor means to asset ownership giving ZIFS banks dual economic goals – to finance and to own. Further, it would appear to contravene another pertinent Qur'ānic edict wherein the borrower drafts the contract [Quran-2:282 (Asad)]. How many borrowers would want their bankers to be permanent partners in ownership? The reward horizon benefitting the bank is simply too deep especially when the PLS borrower may be in a position to conclude that relationship and reduce its managerial obligations.

Khan (1995) points out that a new-comer investor would want a PLS contract because of the risk-sharing advantage. However, the ZIFS banker is not sure about such an entity because of potential Adverse Selection (ADS) problem. According to him, although disputable, even posting collateral does not allow proper differentiation as to investor's bankability. Regardless, one senses that Khan (1995) is struggling with the issue of allocation of funds under ZIFS: what was expected and what is happening, and how the differential may be remedied. Zahir and Hassan (2001) state that "MU contracts may open back door to interest. So, while permissible, it should still be restricted or avoided." While recognizing both MU and PLS, Chapra (1985), and Kahf and Khan (1992), realize that the former is more likely to violate the underlying religious bidding.

Further in the case of Pakistan, according to Khan (1995), the government borrows directly from the public at a very high, tax free interest rate (14%) while also seeking MU financing for some of its purchases. This likely raises expected mark-up rate and depletes incentive for ZIFS banks to explore PLS portfolio options more aggressively. Thus, the use of *Istiṣnāʿ*, although legal by the Letter of the Law, would appear to expand the reach of MU beyond what is sanctioned by the Spirit of the Law. Also, to the extent ZIFS banks use international money market rate (e.g., LIBOR) in determining the mark-up rate, (Zahir and Hassan, 2001), concern arises again as to whether that compromises the underlying ethics of this undertaking, while at the same time over-riding their own economic cost and local

market signal determining that rate. So, MU may be insulated and made artificially more profitable than it really is.

On the other hand, Hassan (2006) argues that the ZIFS entities lag in both technical and allocative efficiency. In his empirical analysis, from the relative point of best practice, ZIFS entities seem to operate at 84% technical efficiency and 73% allocative efficiency. The technical inefficiency is dubbed by him as X-inefficiency arising from managerial, structural, labor related, or other dysfunctions. That is, given the resources deployed regardless of allocative efficiency, they do not function at a level of relative technical efficiency comparable with alternative IFS banks. He thinks that perhaps limiting political appointees and increasing the number of properly trained stewards would mitigate such awkward performance. Regarding allocative inefficiency, Hassan (2006) uses three measurement tools, one of which is the cost minimization rule. Our model, however, will focus on profit maximization rule instead.

In this paper, we will show how ideally a profit maximizing ZIFS bank allocates funds between PLS and MU. We will also explain how an optimal sum deployed in either portfolio is further optimally allocated among multiple portfolio choices within the sector to achieve allocative efficiency. In the process, we will discover ways to regulate the flow of funds in either direction. Explanations are found why MU is dominating PLS in the current lending market.

3. Methodology and Model

Cost minimization and profit maximization are the two different ways of formulating optimization models in Economics and Finance. In measuring the efficiency of a bank, DeYoung and Nolle (1996) argue that cost-based models have the danger of misrepresenting the nature and the extent of inefficiency in banks. Berger and Mester (1997) are also of the opinion that profit maximization is superior to cost minimization to study firm performance as the former takes into account both revenue and cost into consideration. Because of the essential asymmetry to the two portfolio choices of a ZIFS bank that faces uncertain returns from projects undertaken, we formulate the objective function with an expected profit model that takes into account average amounts of investment per project. Also because of the risk differentials, we discount each sector's profit by the risk factor for each of the two possible portfolio choices as justified by Sharpe (1994).

Our objective here is to partition a fixed sum of loanable funds so that the expected rate of profit per unit of risk is equalized between the two choices. The

outcrop of this will be to obtain the optimal sectoral allocations of the total volume of available loanable funds together with the optimal average size of investment and the corresponding number of projects in each sector. Salvaged investment is included in our objective function as a part of the net profit. Also, compared to MU, for PLS, there is typically a delay before cash flow commences. Our objective function addresses this element by including a time variable to measure the opportunity cost that arises in choosing PLS over MU. So, our risk discounted expected profit function becomes:

$$DE(\pi) = \frac{Lb\left\{\frac{P_r r_r}{k_r} + (1 - P_r)S_r\right\}}{\sigma_r} + \frac{L(1-b)\{\frac{P_m r_m}{k_m} + (1-P_m)S_m - \frac{tP_r P_m r_r}{k_r}\}}{\sigma_m} = L\left[\frac{bA}{\sigma_r} + \frac{(1-b)B}{\sigma_m}\right] (1.0)$$

Here,

L = available loanable funds

b = fraction of money deployed as MU investments (sector r), $0 \leq b \leq 1$

(1 – b) = remaining fraction of money deployed as PLS investments (sector m)

P_i = Probability of successful investments in the i^{th} sector, where $0 \leq P_i \leq 1$, (i = r, m)

σ_i = Standard Deviation of number of successful investments in the i^{th} sector

S_i = Fraction per unit of unsuccessful investments in the i^{th} sector that is salvaged, $0 \leq S_i \leq 1$

t = Mean time difference between commencing PLS and MU cash flows, $t \geq 0$, implying PLS taking longer to mature

k_i = Average amount of investment per project in the i^{th} sector, $0 < k_i$

λ_i, δ_i = State (i.e., natural, political, or economic) variable that, respectively, positively and negatively affect the average profitability of the i^{th} sector

$r_i(b, k_i; \lambda_i, \delta_i)$ = Average per project of aggregate cumulative profit on investments in the i^{th} sector, where $r_{rb} < 0$, $r_{mb} > 0$, $r_{ik} > 0$; and $r_{i\lambda} > 0$ and $r_{i\delta} < 0$, where

$r_i = \frac{\sum_{j=1}^{n_i}\sum_{t=1}^{T}(e_{i_{tj}} - c_{i_{tj}})}{n_i}$, and e_{itj} and c_{itj} are earning and cost (includes direct operational costs plus loan/investment outlay), respectively; T = average duration of any contract ($0 \leq T \leq N$, with N = average life-time of project)

$\frac{P_i r_i}{k_i}$ = Risk unadjusted expected average profit per project per unit of average investment outlay per project in the i^{th} sector

$(1 – P_i)S_i$ = Fraction of total outlay of unsuccessful investment in the i^{th} sector that is salvaged

$\frac{tP_r P_m r_r}{k_r}$ = Opportunity cost per unit of successful PLS investment in terms of foregone return on MU investment

$A = \frac{P_r r_r}{k_r} + (1 - P_r)S_r$

$B = \frac{P_m r_m}{k_m} + (1 - P_m)S_m - \frac{tP_r P_m r_r}{k_r}$

$DIF = \frac{A}{\sigma_r} - \frac{B}{\sigma_m}$ = Differential between risk discounted expected profit per unit of MU and PLS outlays, respectively.

Next, we derive the optimum values of the three choice variables (b^*, k_m^* and k_r^*) from the first order conditions from Equation (1.0)[6].

Then, from the first order condition $\frac{\partial DE}{\partial b} = 0$[7], we calculate the optimum value of b^*:

$$b^* = \frac{DIF^* + \frac{\left\{\frac{P_m r_{mb}}{k_m^*} - \frac{tP_r P_m r_{rb}}{k_r^*}\right\}}{\sigma_m}}{\frac{\left\{\frac{P_m r_{mb}}{k_m^*} - \frac{tP_r P_m r_{rb}}{k_r^*}\right\}}{\sigma_m} - \frac{P_r r_{rb}}{k_r^* \sigma_r}} = \frac{N}{D} \qquad (2.0)$$

Since $0 \leq b^* \leq 1$, and $D > 0$, N has to be ≥ 0.

For $b^* \geq 0$, $\quad DIF^* \geq -\frac{\left\{\frac{P_m r_{mb}}{k_m^*} - \frac{tP_r P_m r_{rb}}{k_r^*}\right\}}{\sigma_m}$ (3.0)

Again, for $b^* \leq 1$, $\quad DIF^* \leq -\frac{P_r r_{rb}}{k_r^* \sigma_r}$[8] (4.0)

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[6] $\partial A/\partial b = P_r r_{rb}/k_r < 0$ (i)
$\partial B/\partial b = P_m r_{mb}/k_m - tP_r P_m r_{rb}/k_r > 0$ (ii)
$\partial A/\partial k_r = P_r[k_r r_{rk} - r_r]/k_r^2$ (iii)
$\partial B/\partial k_r = -tP_r P_m[k_r r_{rk} - r_r]/k_r^2$ (iv)
$\partial A/\partial k_m = 0$ (v)
$\partial B/\partial k_m = [P_m(k_m r_{mk} - r_m)]/k_m^2$ (vi)

[7] $L[(A + b\partial A/\partial b)/\sigma_r - \{B - (1 - b)\partial B/\partial b\}/\sigma_m] = 0$
Substituting (i) & (ii) above and simplifying,
$[\{A + bP_r r_{rb}/k_r\}/\sigma_r - \{B - (1 - b)\{P_m r_{mb}/k_m - tP_r P_m r_{rb}/k_r\}\}/\sigma_m] = 0$
Next, substituting for 'A' and 'B' above,
$[\{P_r r_r/k_r + (1 - P_r)S_r + bP_r r_{rb}/k_r\}/\sigma_r - \{P_m r_m/k_m + (1 - P_m)S_m - tP_r P_m r_r/k_r$
$- (1 - b)\{P_m r_{mb}/k_m - tP_r P_m r_{rb}/k_r\}\}/\sigma_m] = 0$
Or $\quad bP_r r_{rb}/k_r/\sigma_r - b\{P_m r_{mb}/k_m - tP_r P_m r_{rb}/k_r\}/\sigma_m = -\{P_r r_r/k_r + (1 - P_r)S_r\}/\sigma_r + \{P_m r_m/k_m + (1 - P_m)S_m - tP_r P_m r_r/k_r - \{P_m r_{mb}/k_m - tP_r P_m r_{rb}/k_r\}\}/\sigma_m$
Or $\quad b = [-\{P_m r_m/k_m + (1 - P_m)S_m - tP_r P_m r_r/k_r - P_m r_{mb}/k_m + tP_r P_m r_{rb}/k_r\}/\sigma_m + \{P_r r_r/k_r + (1 - P_r)S_r\}/\sigma_r] /[-P_r r_{rb}/k_r\sigma_r + P_m r_{mb}/k_m\sigma_m - tP_r P_m r_{rb}/k_r\sigma_m]$

[8] For $N/D \leq 1$, $[DIF^* + \{P_m r_{mb}/k_m^* - tP_r P_m r_{rb}/k_r^*\}/\sigma_m] \leq [-P_r r_{rb}/k_r^*\sigma_r + P_m r_{mb}/k_m^*\sigma_m - tP_r P_m r_{rb}/k_r^*\sigma_m]$

Equations (3.0) and (4.0) determine the upper and lower limits of the risk discounted expected profit per unit differential between PLS and MU investments.

$$-\left[\frac{\left\{\frac{P_m r_{mb}}{k_m^*} - \frac{tP_r P_m r_{rb}}{k_r^*}\right\}}{\sigma_m}\right] \leq DIF^* \leq -\frac{P_r r_{rb}}{k_r^* \sigma_r} \qquad (5.0)$$

When DIF^* is equal to the right-hand side, the risk discounted expected profit per unit from MU investment is large enough to channel all funds into MU. As DIF^* falls short of the right-hand side, b^* decreases, and a combination of MU and PLS investments ($0 < b^* < 1$) becomes profitable. Again, when DIF^* is small enough to equal the negative left-hand side term, $b^* = 0$, and all loanable funds are invested in PLS.

Next, we derive the optimum values of the two choice variables (k_m^* and k_r^*) from the first order conditions from Equation (1.0)[9],

$$\frac{\partial DE}{\partial k_m} = 0$$

Or

$$L(1-b)\frac{\left[\frac{P_m(k_m r_{mk} - r_m)}{k_m^2}\right]}{\sigma_m} = 0$$

Or

$$k_m^* = \frac{r_m}{r_{mk}} \qquad (6.a)$$

We may rewrite Equation (6.a) as average profit per unit of average outlay being equal to the marginal of average profit:

$$\frac{r_m}{k_m^*} = \frac{\partial r_m}{\partial k_m^*}\ ^{10} \qquad (6.b)$$

It may also be rewritten to take an expression of elasticity. Thus,

$$\frac{\partial r_m}{\partial k_m^*} \cdot \frac{k_m^*}{r_m} = 1 \qquad (6.c)$$

Again, using Equation (1.0),

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[9] $\partial A/\partial k_r = P_r[k_r r_{rk} - r_r]/k_r^2$ (i)
$\partial B/\partial k_r = -tP_r P_m[k_r r_{rk} - r_r]/k_r^2$ (ii)
$\partial A/\partial k_m = 0$ (iii)
$\partial B/\partial k_m = [P_m(k_m r_{mk} - r_m)]/k_m^2$ (iv)

[10] Substituting above from (iii) & (iv)

$$\frac{\partial DE}{\partial k_r} = 0$$

Or $$L[\frac{b\frac{(\partial A)}{\partial k_r}}{\sigma_r} + \frac{(1-b)\frac{\partial B}{\partial k_r}}{\sigma_m}] = 0$$

Or $$k_r^* = \frac{r_r}{r_{rk}} \qquad (7.a)$$

We may rewrite Equation (7.a) as average profit per unit of average outlay being equal to the marginal of average profit:

$$\frac{r_r}{k_r^*} = \frac{\partial r_r}{\partial k_r^*} \quad [11] \qquad (7.b)$$

In elasticity term, we get,

$$\frac{\partial r_r}{\partial k_r^*} \cdot \frac{k_r^*}{r_r} = 1 \qquad (7.c)$$

The expected signs associated with the change in b^* with respect to t, S_m, S_r, P_m, σ_m, P_r, and σ_r are given below. However, only the first three inequalities [(8.0), (9.0) and (10.0)] are unambiguously proven to be as anticipated. The rest of the signs are not definitive as expected because the magnitudes of certain variables are unknown:

$$\partial b^*/\partial t > 0 \quad [12] \qquad (8.0)$$
$$\partial b^*/\partial S_m < 0 \quad [13] \qquad (9.0)$$
$$\partial b^*/\partial S_r > 0 \quad [14] \qquad (10.0)$$
$$\partial b^*/\partial P_m < 0 \quad [15] \qquad (11.0)$$
$$\partial b^*/\partial \sigma_m > 0 \quad [16] \qquad (12.0)$$
$$\partial b^*/\partial P_r > 0 \quad [17] \qquad (13.0)$$
$$\partial b^*/\partial \sigma_r < 0 \quad [18] \qquad (14.0)$$

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[11] Substituting (i) & (ii)

[12] $\partial b^*/\partial t = [D(-P_mP_rr_{rb}/k_r\sigma_m + P_rP_mr_r/k_r\sigma_m) + NP_mP_rr_{rb}/k_r\sigma_m]/D^2 = [(N - D)P_mP_rr_{rb}r_{rk}/r_r\sigma_m + DP_rP_mr_{rk}/\sigma_m]/D^2 > 0$

[13] $\{-(1 - P_m)/\sigma_m\}/D < 0$

[14] $\{(1 - P_r)/\sigma_r\}/D > 0$

[15] $[D\{-r_m/k_m\sigma_m + S_m/\sigma_m + tP_rr_r/k_r\sigma_m + r_{mb}/k_m\sigma_m - tP_rr_{rb}/k_r\sigma_m\} - N\{-tP_rr_{rb}/k_r\sigma_m + r_{mb}/k_m\sigma_m\}]/D^2$
$= [(D - N)(r_{mb}/k_m\sigma_m - tP_rr_{rb}/k_r\sigma_m) - D(r_m/k_m\sigma_m - S_m/\sigma_m - tP_rr_r/k_r\sigma_m)]/D^2 > 0$, when $[r_m/k_m - S_m - tP_rr_r/k_r] < 0$

[16] $[D\{P_mr_m/k_m + (1 - P_m)S_m - tP_rP_mr_r/k_r\}/\sigma_m^2 - (D + N)\{P_mr_{mb}/k_m - tP_mP_rr_{rb}/k_r)\}/\sigma_m^2\}]/D^2 > 0$, when $[P_mr_m/k_m + (1 - P_m)S_m - tP_rP_mr_r/k_r > (1 + b)(P_mr_{mb}/k_m - tP_mP_rr_{rb}/k_r)$

[17] $[D\{(r_r/k_r - S_r)/\sigma_r + tP_mr_r/k_r\sigma_m - tP_mr_{rb}/k_r\sigma_m\} + N\{r_{rb}/k_r\sigma_r + tP_mr_{rb}/k_r\sigma_m\}]/D^2 > 0$, when $\{(r_r/k_r - S_r)/\sigma_r + tP_mr_r/k_r\sigma_m - tP_mr_{rb}/k_r\sigma_m\} > -b\{r_{rb}/k_r\sigma_r + tP_mr_{rb}/k_r\sigma_m\}$

[18] $[-D\{P_rr_r/k_r + (1 - P_r)S_r)\}/(\sigma_r)^2\} - N\{P_rr_{rb}/k_r\}/(\sigma_r)^2\}]/D^2 < 0$, when $[P_rr_r/k_r + (1 - P_r)S_r)] > -b[P_rr_{rb}/k_r]$

4. Results

Our formulation of the objective function is based on profit maximization, under perfect competition, using average revenue and average cost which necessarily deviates from the standard formulation based on total revenue and total cost. Our formulation is suitable in this case where neither MU nor PLS investments are homogeneous like 'Q' in [$\pi = PQ - C(Q)$] model. The adoption of average values for them as choice variables allows a form of homogeneity which makes formulation of the objective function easier. However, the advantage of using the average perspective for intra-sectoral allocation determination lies in the fact that the ZIFS bank will have wide latitude to vary the amounts of MU and PLS investments around the average amount. This converges with the practical reality of how any such bank generally operates. Although our derived rules are unique, they pose a challenge to interpret and use.

Equation (2.0) determines optimal inter-sectoral allocation, while equations (6.a), (7.a), (6.b), and (7.b), (6.c.) and (7.c) give symmetric results that suggest optimal intra-sectoral allocation. Equations (6.a) and (7.a) give us the optimal average MU and PLS investments that a ZIFS institute should put out, i.e., k_r^* and k_m^*. Given the optimal inter-sectoral allocation b^*, and that $k_r^* = \frac{b^* L}{n_r^*}$ and $k_m^* = \frac{(1-b^*)L}{n_m^*}$, we are simultaneously able to derive the optimal number of projects per sector, i.e., n_r^* and n_m^*. At equilibrium, according to equations (6.b) and (7.b), k_r^* and k_m^* are such that the average profit per unit of average outlay and the marginal of the average profit with respect to average outlay are equal.

Also at equilibrium, according to equations (6.c) and (7.c), another alternative explanation says that k_i^* is such that the elasticity of net average profit per project with respect to average amount of investment is equal to one (1) in both the sectors. In other words, with an optimal allocation, the percentage change in total outlay in MU/PLS must equal the percentage change in corresponding total profit. Thus, given the available loanable funds in a sector, the optimal number of projects within each sector should be such that the unit elastic point of this elasticity measure is reached. Note that uncertainty and risk play no role in the solution set identifying the optimal intra-sectoral allocation of average investment.

The inequality (5.0) shows that the amount of investment in MU/PLS depends on DIF*, the risk discounted expected profit per unit differential between PLS and MU investments. The lower is the DIF*, the lower is PLS investment. The conditions under which MU funding will dominate PLS funding include: high P_r, low P_m, low σ_r, high σ_m, high t and low S_m. Obviously, more funds will be in PLS in a region where P_m is significantly higher than P_r. Policy makers can increase S_m through subsidies (i.e. tax break for losses in PLS investments) or insurance to protects against losses in order to attract more funds into PLS. Also, ways to reduce σ_m and t will reduce reliance on MU and move funds to PLS if so desired. Effects of policy changes are explained in detailed below.

Effects of Policy Changes

To see the impact of a change in state variables (λ or δ) such as subsidies, taxes, tariffs, exchange rate, quotas, regulations, utility or industrial/export zone provision, minimum wages, better forecasting, improvement in public sector training of workforce, enforcement of the Rule of Law, etc., we will consider changes in b^*, k_m^* and k_r^* as outcomes of changes in relevant 'λ_i' or 'δ_i' (i = r, m) Thus, increases in λ_i and δ_i are understood to, accordingly, increase and decrease the profitability of projects supported by type-i loans. Since δ_i impacts in exactly the opposite way as does λ_i, so only derivatives with respect to the latter will be taken.

$$\frac{\partial b^*}{\partial \lambda_m} = \frac{-\frac{P_m r_{m\lambda}}{k_m^* \sigma_m}}{\frac{\left\{\frac{P_m r_{mb}}{k_m^*} - \frac{tP_r P_m r_{rb}}{k_r^*}\right\}}{\sigma_m} - \frac{P_r r_{rb}}{k_r^* \sigma_r}} < 0 \qquad (15.0)$$

$$\frac{\partial b^*}{\partial \lambda_r} = \frac{\frac{P_r r_{r\lambda}}{k_r^* \sigma_r} + \frac{tP_r P_m r_{r\lambda}}{k_r^* \sigma_m}}{\frac{\left\{\frac{P_m r_{mb}}{k_m^*} - \frac{tP_r P_m r_{rb}}{k_r^*}\right\}}{\sigma_m} - \frac{P_r r_{rb}}{k_r^* \sigma_r}} > 0 \qquad (16.0)$$

$$\frac{\partial k_m^*}{\partial \lambda_m} = \frac{r_{m\lambda}}{r_{mk}} > 0 \qquad (17.0)$$

$$\frac{\partial k_r^*}{\partial \lambda_r} = \frac{r_{r\lambda}}{r_{rk}} > 0 \qquad (18.0)$$

Equation (15.0) states that policies that increase the profitability of the PLS projects (subsidies, etc.) will reallocate funds from MU to PLS investments. Equation (17.0) indicates that an increase in the same variables additionally increases the average amount of investment per project in that sector, implying also that the optimal number of projects supported with loans will diminish. Equations (16.0) and (18.0) show symmetric results for MU investments.

For example, if in an economy wages and benefits are raised in the PLS sector (owing to subsidized lifestyle of citizen civilian population capable of working in this sector thereby raising expectation incommensurate with their potential productivity), while they are depressed in the MU sector (owing to the availability of low paid immigrant, blue collar and pink collar labor), profitability will decrease in the former sector and increase in the latter. According to equations (15.0 and 16.0), funds will be driven from PLS sector to MU. Of course, this is in the context of nascent industrial undertakings since established firms are likely to expand using their own resources.

Further, in the presence of trade surplus and low population pressure, commodity trade may be used extensively to substitute for goods produced domestically. Business in such products is readily amenable to MU loans (since prior-to-lending physical possession is easy) and when tangible profit flow is ignored as a criterion not to provide such loans. So, with limited drive for import substitution, MU is strengthened many folds.

Below, we check the cross-effects of λ_m (λ_r) on k_r^* (k_m^*). From Equations (6.a) and (7.a), respectively, we calculate $\frac{\partial k_m^*}{\partial b} = \frac{r_{mb}}{r_{mk}} > 0$ and $\frac{\partial k_r^*}{\partial b} = \frac{r_{rb}}{r_{rk}} < 0$. So,

$$\frac{\partial k_m^*}{\partial \lambda_r} = \frac{\partial k_m^*}{\partial b} \cdot \frac{\partial b}{\partial \lambda_r} > 0 \qquad (19.0)$$

$$\frac{\partial k_r^*}{\partial \lambda_m} = \frac{\partial k_r^*}{\partial b} \cdot \frac{\partial b}{\partial \lambda_m} > 0 \qquad (20.0)$$

In other words, the effect of policies that increase profitability in PLS (MU) sector is such that optimal average outlay in the counterpart MU (PLS) sector is increased. Interestingly, these symmetric and positive cross-reactions are the same as those in equations (17.0) and (18.0). It can also be shown that policies to increase profitability in PLS (MU) sector will also increase the average profit per project in the other sector, i.e., $\frac{\partial r_i}{\partial \lambda_j}$ is positive (i, j = r, m; i ≠ j)[19]

The effects of sectorial profitability, i.e., favorable change in λ_m and λ_r, (δ_m or δ_r), respectively, on n_m^* ($= \frac{(1-b^*)L}{k_m^*}$) and n_r^* ($= \frac{b^*L}{k_r^*}$) are captured below. While optimal fund allocation decreases [equations (15.0) and (16.0)] and optimal average outlay

[19] $(\partial r_i/\partial \lambda_j) = (\partial r_i/\partial k_i)(\partial k_i/\partial \lambda_j) > 0$ (i, j = r, m; i ≠ j) (+) (+)

increases [equations (19.0) and (20.0)] with increased cross-profitability in the other sector, we expect and show that the optimal number of outlays also fall.

$$\frac{\partial n_m^*}{\partial \lambda_r} = \frac{-Lk_m^*\frac{\partial b}{\partial \lambda_r} - (1-b)L\frac{\partial k_m^*}{\partial \lambda_r}}{(k_m^*)^2} < 0 \qquad (21.0)$$

$$\frac{\partial n_r^*}{\partial \lambda_m} = \frac{Lk_r^*\frac{\partial b}{\partial \lambda_m} - bL\frac{\partial k_r^*}{\partial \lambda_m}}{(k_r^*)^2} < 0 \qquad (22.0)$$

Finally, we note that when more money flows into a sector [equations (15.0) and (16.0)], an increase in the optimal average outlay [equations (17.0) and (18.0)] may be accompanied by either an increase or a decrease in the optimal number of outlay:

$$\frac{\partial n_m^*}{\partial \lambda_m} = \frac{-Lk_m^*\frac{\partial b}{\partial \lambda_m} - (1-b)L\frac{\partial k_m^*}{\partial \lambda_m}}{(k_m^*)^2} < 0 \text{ or } > 0 \qquad (23.0)$$

$$\frac{\partial n_r^*}{\partial \lambda_r} = \frac{Lk_r^*\frac{\partial b}{\partial \lambda_r} - bL\frac{\partial k_r^*}{\partial \lambda_r}}{(k_r^*)^2} < 0 \text{ or } > 0 \qquad (24.0)$$

As to the impact of a change in δ_i, it will be exactly opposite the change affected by a changing λ_i.

5. Conclusion

This paper sought to better enunciate the problem of loanable funds allocation faced by ZIFS entities. Studies, to date, in this area have focused on the ethics and the normative expectations, empirical evaluation, analysis of the validity of various financial instruments adopted for this end, and theories to explain some of the characteristics or behaviors exhibited by the various players. Some of the concerns have included: allocative efficiency as well as the apparent aversion to engage in PLS.

Since both of the above issues pertain to allocation, we formulate a risk discounted, expected profit function with two possible portfolios choices. In the formulation, we use Proportional distributional parameters, total profit function in terms of averages, and differential time to accrual of initial profit to determine the division of loanable funds between the two portfolio choices as well as the average outlay sums within each portfolio type. Our results on inter-sectoral allocation of loanable funds and the nature of intra-sectoral allocation of designated monies clearly side with anticipated relationship, whether it is with the distributional parameters, time dimension, profitability, etc.

In sanctioning MU sector under ZIFS banking, there is no specification about what constitutes MU rate or how it should be arrived at. Also, in that very process,

there has been a lack of consideration that both the banks and the public sector could drag the definition of MU to expand to such an extent that the domain of PLS could practically disappear.

Thus, for example, a more formal, comprehensive consideration is needed to determine the criteria under which a credit request could be converted to MU form. The criteria could be: whether there is a tangible profit flow in the use of the loan, whether the debt is for durable consumption goods, trade, small businesses or public goods, and the lifetime of the debt. Some of these elements could lead to specifying the amount of the credit request that could be served through MU. On the other hand, understanding is needed as to what extent MU may be used for deficit financing the Government or for acquisition of public goods, especially when it may opportunistically resort to both interest based and MU debts, thereby crowding out private borrowers with significant consequence for the PLS sector.

However, some of the contradictions that researchers in the field are trying to explain and resolve, including the likely demand diminishing, permanent and fixed partnership under PLS, may not be amenable to a quick fix if only the interpretation of textually sanctioned options are debated without considering the underlying spirit of equity as well as other related Qur'ānic verse(s). Further, the apparent anomalies or overreach allows for cynicism on the part of critics as well as making room for competing secular banks to exploit the needs of well-meaning but gullible depositors and credit seekers thereby undermining the basic proposition.

Regardless, we expect our results to serve several ends: clarify the overall nature of the problem, give it a more scientific footing by removing it from the arena of invocations and assertions, explain what drives the allocation, and give a proper technical perspective to ethicists, researchers, policy makers, bankers, borrowers and investors. Now, our perspective has been from the supply side. Any deficiency in the current system emanating from the demand side demands a separate study.

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